Stars Test #1 Answer Key

1. a. $E = \frac{m}{\alpha} = \frac{1.6726\times10^{-27} kg}{5.5\times10^{-16} kg/J} = 3.04\times10^{-12} J$

b. $E = mc^2 = 1.6726\times10^{-27}(2.9979\times10^{8})^2 = 1.503\times10^{-10} J$
So the energy produced in our universe is nearly 50 times greater…

c. Solve the central pressure for mass

$$P_c > \frac{GM^2}{8\pi R^4}$$

$$10^{13} = \frac{GM^2}{8\pi R^4} = \frac{6.6726\times10^{-11} M^2}{8\pi(6.9598\times10^{8})^4}$$

$$M^2 = 8.841\times10^{59}$$

$$M = 9.403\times10^{29} kg$$

d. 1 solar mass = $1.99\text{x}10^{30}$ kg, so the amount of mass lost would be
$1.99\times10^{30} - 9.403\times10^{29} = 1.05\times10^{30} kg$

e. Energy given off by 1 solar mass star during life =

$$E = \frac{m}{\alpha} = \frac{1.05\times10^{30} kg}{5.5\times10^{-16} kg/J} = 1.91\times10^{45} J$$

f. Time = Energy/Luminosity =1.91x 10^{45}J/8 x 10^{27} J/s= $2.39\text{x}10^{17}$ seconds.
This is also the equivalent of 7.5 billion years.

g. The lowest mass in solar masses is $9.403\text{x}10^{29}/1.99\text{x}10^{30}$=0.473 solar masses. Putting that into the L-M relationship gives –

$$L = M^{2.5} = (.473)^{2.5} = 0.154$$

So the luminosity of the lowest mass star is 0.154 times the luminosity of the sun or 0.154 x 8 x 10^{27} = 1.23 x10^{27} J/s.

h. $L = M^{2.5} = (25)^{2.5} = 3125$ times the luminosity of the “Sun”, or 3125 x 8 x 10^{27} J/s = $2.5\text{x}10^{31}$ J/s.

i. The 25 solar mass star uses almost all of its mass except for the amount found in part c. That means (25 x $1.99\text{x}10^{30}$kg) – $9.403\text{x}10^{29}$ kg = $4.88\text{x}10^{31}$ kg is used. This produces

$$E = \frac{m}{\alpha} = \frac{4.88\times10^{31} kg}{5.5\times10^{-16} kg/J} = 8.87\times10^{46} J$$

j. Basically take the result from part “i” and divide it by the result from part “h”.
$8.87\text{x}10^{46}$ / $2.5\text{x}10^{31}$ = $3.55\text{x}10^{15}$ s = about 112 million years.

2. Opacity goes up. This hinders the flow of energy outwards. This will cause an increase in the pressure and temperature of the star. Eventually the buildup in pressure will result in the star

expanding outwards. The expansion will continue until the material is thin enough so that the radiation doesn't interact with it anymore, and the energy can escape freely. The increase in temperature could change the energy generation rate in the star, but there is also a decrease in density which could lower it as well – odds are the temperature increase would be more significant so more energy is produced and the star would be in even worse shape (even more energy is trapped).

3 Convection is really a 3-D event involving the changing of mass located within a star, but in a way that doesn't alter the distribution significantly. To monitor the motion of all of these different blobs in 3 dimensions would be crazy. There is no accepted method to accurately approximate how convention alters a star since it happens in such a short time compared to other changes in a star's life. Even though we know convection actually does happen in a star, we can usually gloss over the changes in material re-distribution – which is actually the second part of the question. Very strong convection can re-distribute different chemical mixes throughout the star by bringing material from the center (generally more metal enhanced) to the surface.

4. The four formula with the "d" removed –

$$\frac{P}{R} = -\frac{\rho GM}{R^2}$$

$$\frac{M}{R} = 4\pi R^2 \rho$$

$$\frac{T}{R} = -\frac{3\kappa\rho F}{4acT^3 4\pi R^2}$$

$$P = \frac{\rho \Re T}{\mu}$$

Move the R values on the left to the right in the first three equations, and simplify

$$P = -\frac{\rho GM}{R}$$

$$M = 4\pi R^3 \rho$$

$$T = -\frac{3\kappa\rho F}{4acT^3 4\pi R}$$

$$P = \frac{\rho \Re T}{\mu}$$

Combine the first and fourth to get rid of the Pressure

$$P = -\frac{\rho GM}{R} = \frac{\rho \Re T}{\mu}$$

Solve this for T (densities cancel out)

$$T = -\frac{\mu GM}{\Re R}$$

Rewrite the radiative transfer formula with T values on the left

$$T^4 = -\frac{3\kappa\rho F}{4ac4\pi R}$$

Take the previous "T" formula – take both sides to the fourth power and set it equal to the radiative transfer formula

$$\frac{\mu^4 G^4 M^4}{\Re^4 R^4} = -\frac{3\kappa\rho F}{16ac\pi R}$$

Solve the conservation of mass formula for density and substitute that into the above.

$$\rho = \frac{M}{4\pi R^3}$$

$$\frac{\mu^4 G^4 M^4}{\Re^r R^4} = -\frac{3\kappa F M}{16ac\pi R 4\pi R^3}$$

R^4 on both sides cancels out.
One M on each side cancels out.

Final formula solved for F (or L) and M

$$M^3 = -F\,\frac{\Re^4 3\kappa}{64ac\pi^2\mu^4 G^4}$$

$$F = L = -M^3\,\frac{64ac\pi^2\mu^4 G^4}{3\kappa\Re^4}$$

This is pretty similar in that the luminosity goes as the mass to the third power – one of the options for stars.

5a. $1/\mu_e$ = 1+0+0, μ_e**=1**
b. 1/ μ_e =.5+.5x.5+0=.75, μ_e**=1.33**
c. 1/ μ_e =0+.5+0, μ_e**=2**
d. 1/ μ_e =0+.5x.5+.5x26/56)=0.482, μ_e =**2.07**
e. 1/ μ_e =0+0+26/56=0.46, μ_e **=2.15**
f. 1/ μ_e =0+0+82/206=0.398, μ_e **=2.51**

g. The relationship is $M_c = 5.83/(\mu_e)^2$ solar masses. So the value of μ_e increases in parts a-f as the composition goes to more massive atoms, which would decrease the Chandrasekhar mass. It would range from 5.83 for the first case to 0.925 for the all lead case. Generally it is something in the middle, like for part c, which gives a mass of 1.46 solar masses!

6. Basically on the top – high, high luminosity areas. Of course when the mass of the star is low and it is high luminosity (like in a late evolutionary stage for an intermediate mass star), they are found in the upper right corner of the HR diagram, and there the luminosity values are way larger than the corresponding mass.