

Black Holes

What is the other option for a massive star's final demise? A black hole! Black holes are not anything too exotic or unusual when viewed in their simplest form. They are just objects that have really high gravitational pulls (sort of like "gravity gone wild"). What is the situation where you have "too much gravity"?

The easiest way of looking at the limits of gravity is looking at the definition of the escape velocity – how fast you need to travel to escape the gravitational pull of an object. This goes way back to equation 2-20. This is simply –

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Now let's see how far we can go with that. The fastest known velocity is the speed of light, c , so let's use that as the velocity. In that case

$$c = \sqrt{\frac{2GM_{BH}}{R_{BH}}}$$

$$c^2 = \frac{2GM_{BH}}{R_{BH}}$$

$$R_{BH} = \frac{2GM_{BH}}{c^2}$$

The final result (equation 8-1) defines the *Schwarzschild radius* for the black hole. You can view this as the size of the black hole, though it is technically the distance from a black hole that you can escape from so long as you are traveling at the speed of light. Anything closer is dead. And even if you are at the Schwarzschild radius you are also dead since you can't travel at the speed of light to escape.

8-1

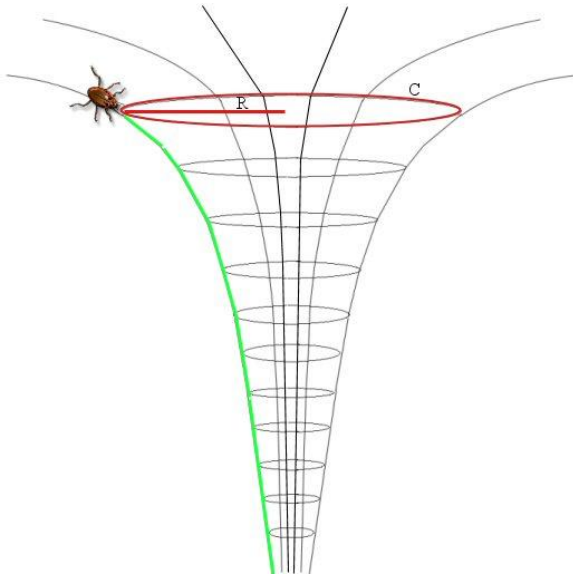
Another way to think of what this formula represents is as a compression threshold. If you crush something down to the size of its Schwarzschild radius, it would become a black hole.

So does the Schwarzschild radius really define the "size" of a black hole, or your actual distance from a black hole? No.

General relativity tells us that space is distorted in regions that contain mass, and the more concentrated the mass, and the greater the mass, the greater the distortion. So the space near a black hole is very warped as is shown in the illustration on the next page. While you could not physically see the warping of the space near the black hole, you would be influenced by its effect on your motion or your measurements of time, and distance in that area. You are used to measuring distances and areas based upon a non-warped space – your basic geometry classes probably used flat pieces of paper to show triangles and circles, not warped sheets of paper. When you deal with black holes, the space is warped so you can't simply measure the sizes as if the locations are in flat space.

The Schwarzschild radius is generally defined in Euclidean space (non-warped), so there is the belief that the black hole is only that far away (the line marked as "R"). But looking at the image, you can see that the actual distance is much greater due to the warping of space in this area, and the actual radius is denoted by the green line in the

diagram (as measured from the bug's location). So the actual distance to the black hole isn't really the Schwarzschild radius, so that isn't a very useful term in helping us to determine features around the black hole.



Rather than using R_{sch} to describe features around black holes, we'll instead use the circumference of the black hole as defined by the Schwarzschild radius, since that isn't influenced by the warped space.

This value is given by

$$C_{BH} = 2\pi R_{sch} = \frac{4\pi GM}{c^2} \quad 8-2$$

In order to safely investigate the black hole you don't actually want to go to the distance of the circumference (since you'll die). Instead we'll set up a location outside of the black hole that's a safe distance away. And to keep things

consistent we won't use the distance from the black hole to define that location (since it would also involve warped space) but we'll use the circumference of that location.

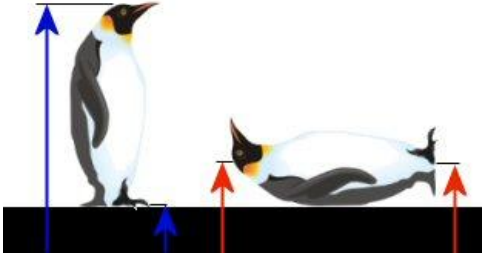
So let's define an orbital "distance" from the black hole where we can safely observe it and that is at the distance R_o . This would have a corresponding circumference of $C_o = 2\pi R_o$. Just like the Schwarzschild radius, R_o is the Euclidean space distance for the orbit, and C_o is the orbital circumference. And if you want to remain safe and alive you'll have to have $C_o > C_{BH}$.

Okay, so now you're in orbit about a black hole. That means you have to be going around the black hole at a particular pace so that you maintain that distance – not too fast or you'll fly off away from the black hole and not too slow or you'll fall in. You have a specific orbital period (P) that is stable.

The mass of the black hole can be measured safely from our location by using a modified version of Kepler's third law

$$M_{BH} = \frac{C_o^3}{2\pi GP^2} \quad 8-3$$

While you might be safe at the orbital distance you picked so you don't fall into the black hole, there are other things to be concerned about. You are also going to have to watch out for the tidal forces that are at work in this area. Tides are basically a difference in gravitational accelerations on an object, such as how much is one part of you being tugged by the earth versus another part. The relative distances between those parts will help determine the tidal forces that are being experienced.



For example, the penguin on the left will feel a difference in gravitational acceleration between its head and its feet, since they are at different distances from the center of the Earth. The penguin on the right would not experience any difference in the amount of acceleration felt by its feet and head since both are roughly at the same distance from the center of the Earth.

The difference in gravitational acceleration due to gravity can be determined based upon the size of our orbit – since that’s what we would feel in our space ship.

$$\Delta a = \frac{16\pi^3 GLM_{BH}}{C_o^3} \tag{8-4}$$

where the difference in acceleration, Δa , depends on the length/height of the object (L), the mass of the black hole and the circumference of our orbit. The length of the object should be measured radially from the black hole – sort of like the length for the penguin on the left (its height) would be the value for L .

What if you were at an orbital distance/circumference that was based upon the mass of the black hole? For example, you could say that you want to be at an orbital circumference that is 5 times or 10 times or 100 times the black hole circumference (defined by equation 8-2). In that case you can replace the value of C_o with a relation based upon the black hole mass, and you’d get the following –

$$\Delta a \propto \frac{M_{BH}}{C_o^3} \propto \frac{M_{BH}}{C_{BH}^3} \propto \frac{M_{BH}}{M_{BH}^3} \propto \frac{1}{M_{BH}^2}$$

This indicates that the gravitational acceleration difference (“tidal pull”) would depend inversely upon the mass of the black hole, so the largest mass black holes have the smallest tidal effects. Put simply, small mass black holes are more dangerous than large black holes when it comes to tidal forces. In fact you could have situations where large mass black holes would not even cause you any stress or strain near the location of the C_{BH} . So if you have a choice, go towards a really high mass black hole.



I should also mention that not only do you get to be stretched by the tidal forces, but you are also squeezed as you get closer to a black hole – you get narrower. This is known as spaghettification. Really, that’s what it is called. The image at left shows an idealized version of this process (not to scale). Longer and narrower. Again, this is due to the changes in space as you get closer to the black hole.



There are other effects on what also happens in the area of a black hole, including how light behaves. Light that is going towards a black hole will become blueshifted (wavelengths become shorter, higher energy), while light

moving away from it would become redshifted (longer wavelengths, lower energy). So if you are near a black hole you'll see light coming towards you appearing bluer by the factor

$$\lambda_{observed} = \lambda_{original} \sqrt{1 - \frac{C_{BH}}{C_o}} \quad 8-5a$$

And the corresponding redshift is given by

$$\lambda_{observed} = \frac{\lambda_{original}}{\sqrt{1 - \frac{C_{BH}}{C_o}}} \quad 8-5b$$

It is interesting to note what happens when your orbit gets close to the circumference of the black hole. In that case the stuff under the square root becomes 0, so in equation 8-5a that goes to 0 (the light is blue shifted to a wavelength of no size), while in the other direction it goes to infinity (infinitely long wavelengths). So if you watch a light source move towards a black hole from your safe space ship, the light it emits gets redshifted to infinity when it gets to the C_{BH} . Technically you'll not be able to pick it up once it has a wavelength beyond the limits of your technology. It is worth mentioning that as you go towards the black hole and you see distant objects looking bluer and bluer, eventually that light will get to dangerous wavelengths – ultraviolet, x-ray and gamma-ray. So make sure you are protected from the light getting blue shifted as it goes towards a black hole.

And what about time?

The rules say that clocks that are in a gravitational potential (field) run slower than if they are not in a gravitational potential. Let's say you have an eager volunteer who will go towards the black hole with a clock. You will stay safely far away from the black hole. The following relation defines how you would measure the passage of time that your volunteer experiences near the black hole –

$$t_{near} = t_{far} \sqrt{1 - \frac{C_{BH}}{C_o}} \quad 8-6a$$

Where t_{far} is the amount of time passing that you measure, and t_{near} is the time you measure that your volunteer is experiencing at the location near the black hole (C_o). So if you are far from a black hole, the $t_{near} = t_{far}$. But as your friend gets closer to the black hole, the value under the radical gets very small, and the passage of time near the black hole is very small compared to the time that you measure. So it seems that the person near the black hole is moving in slow motion. The effect gets more pronounced (they appear to move even slower) as they get closer to the black hole. And as your victim, uh, volunteer gets really close to C_{BH} , the amount of time that passes as you see it would be observed to be very, very small (since the stuff under the radical gets to be very, very small).

What does the volunteer see? They would not think that anything unusual is going on since all time keepers such as pulse rates and electrical impulses in the brain would also slow down. So the volunteer doesn't get the sense that their own time is unusual – their clock appears normal to them.

Actually the above formula only works if the volunteer isn't actually moving around the black hole. That's pretty unrealistic. For the situation where the volunteer is in orbit about the black hole the formula becomes

$$t_{near} = t_{far} \sqrt{1 - \frac{3C_{BH}}{2C_o}} \quad 8-6b$$

This formula is generally better since odds are the volunteer can't just stand still near a black hole – unless they have a really good rocket to counteract its pull, but that would be too expensive.

Space-Time

Why do all of these peculiar things happen? It has to do with the way that space is distorted by mass and how we measure time and space (distance). Since space is warped by mass, there is an influence on the measurements of space-time due to this distortion. And of course we have to use the term space-time since space and time are interconnected – they are not separable. Generally this is the problem most people have with relativity. Technically the slow down caused by gravity can be measured even on the earth. A clock on the floor and a clock on the ceiling would not keep the same time since they are in a different gravitational field (at different distances from the center of the earth). Obviously we don't see such difference in times since the effect on the earth is not very significant, but it is measureable in other situations. Space and time are linked.

Let's say you have a perspective that there is just "space". Then the difference in location between two points in your basic three dimensional universe is defined by something like

$$ds^2 = dx^2 + dy^2 + dz^2$$

Or something like that for a 3 dimensional space defined by dx , dy , and dz . But with space and time interconnected you don't just measure "distances" between locations but space-time events between two points – or the "metric" of space-time is given by

$$ds^2 = -c^2 dt^2 + (dx^2 + dy^2 + dz^2) \quad 8-7a$$

All of these have " d " since they are measures of small intervals – need that to retain accuracy. ds is not only a representation of distance but is an interval in space-time, and that's why there is also a time component in the relation. And the time component is negative because the event that is seen happens at a distance away (dx, dy, dz) and in the past before you see it.

Most of the time equation 8-7a is not given in Cartesian coordinates, but in spherical coordinates (r, θ, ϕ), so you actually have –

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad 8-7b$$

The above relations 8-7a, 8-7b, would only be valid if there aren't strong distortions of space-time due to the influence of mass, so that space and time are both distorted according to the following metric – also called the Schwarzschild metric

$$ds^2 = -c^2 dt^2 \left(1 - \frac{C_{BH}}{C_o}\right) + \frac{dr^2}{\left(1 - \frac{C_{BH}}{C_o}\right)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad 8-8$$

This basically says that whatever you measure, be it time or distance, it will be influenced by the distortion of space-time by mass.

Other things that we measure or define are also influenced by the distortion of space-time, so all of those other formulae that we went over previously for stellar characteristics are now totally screwed up under the conditions of spatial distortion. For example, the formula for hydrostatic equilibrium –

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2}$$

Would be something like

$$\frac{dP}{dr} = -G \frac{\left(\rho + \frac{P}{c^2}\right) \left(M + \frac{4\pi r^3 P}{c^2}\right)}{r \left(r - \frac{2GM}{c^2}\right)}$$

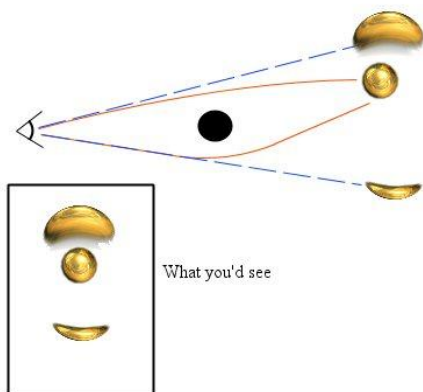
Basically there are a lot factors of c^2 and other terms that pop up all over the place, and we don't really want to look at all of these, do we? We won't be dealing with this stuff.

Traveling to a Black Hole

So what really happens to someone who goes into a black hole? Let's take a trip to a 35 M_\odot black hole. This is a good size since the Schwarzschild radius is about 100 km, and that's an easy number to deal with. The value for C_o is around 650,000 km.

And let's assume that you're about 2 meters tall.

At a distance of about 100 Schwarzschild radii (10,000 km away) from the black hole you have a tidal force between your head and feet equal to about 1 g. So you're probably feeling that, but it isn't killing you.



As you get closer you'll start to notice the influence of gravitational lensing – or the distortion of images caused by the space distortions of the black hole. If there was a galaxy or other form located along your line of sight towards the black hole (but behind it), the image of the galaxy would get distorted as you view it. Gravitational lensing is a very important tool used to determine the masses and distances of objects, since the form of the distortion depends upon the relative distances of the object from the observer and the mass of the lensing object (the black hole

in this case). There is also the angle of alignment of the objects at play as well.

Let's say you have the arrangement pictured above, a black hole (colored black), you to the far left and a spherical object on the right. The light from that object should travel in a straight path but the black hole curves the paths, some of which are redirected towards you. In this case light that would normally not go anywhere near you gets redirected into your direction and you see two warped images of the spherical object – odds are you will not see any normal image of the sphere, I'm just including it here to sort of show its relative placement to the blobs that are also produced.

As you get closer to the black hole, your view of the universe will change in other ways as well. Light coming in from all directions is bent around by the warped space and can appear to get funneled into the black hole, and this would include light that normally wouldn't come near you. The end result is that you see objects in more directions than you are used to seeing. Your field of vision under normal circumstances is around 180 degrees, but as you get closer to the black hole, the light from objects that are beyond this range starts getting funneled towards you. You will start seeing things which are actually behind you, appearing in front of you.

You would also be forced to view the universe in only one direction. The funnel of light will get narrower and narrower as you near the black hole. But it will be difficult to see anything clearly due to the insanely large distortions that are near the black hole.

The closest you can get to the black hole and have a stable orbit is at 3 Schwarzschild radii (310 km away). This would also be the area that you would find any other matter falling towards the black hole to be piling up. Generally you'd find the inner edge of an accretion disk at about this location.

At a distance of 3 Schwarzschild radii, you'd have a very interesting perspective. First of all your orbit is only around 1900 km in circumference and you'd be going around at about 60% the speed of light. One orbit would take only 0.01 seconds as you measure it in your spaceship. The orbital velocity would be directly proportional to the black hole mass and your distance. At this distance it wouldn't really matter how fast you are moving, since the tidal forces between your head and feet is around 63,000 g.

At a distance of 1.5 x Schwarzschild radius a stable orbit is only possible if you travel at the speed of light, so that means you can't do it (yikes). This location is also called the *photon sphere*, since light (with just the right trajectory) could remain in a stable orbit around the black hole. Anything closer than $1.5 R_{\text{BH}}$ would have to travel at a speed greater than light to stay in orbit, and that's not possible. If you want to stay alive within the photon sphere, you'd have to use your retro-rockets to fight the pull of gravity as well as use your rockets to orbit. This will end up being a very costly, and a eco-unfriendly journey (not very fuel efficient).

If you get down to the Schwarzschild radius, you are at the last location that you could be observed by someone far from the black hole. But it would be pretty difficult to detect

you since any light you give off would be redshifted to such long wavelengths it would be hard to pick up with current technology. Also the pace of your observed motion would be so small it (the time distortion effect) that it would be difficult to see you moving at all even over a long time. Of course in your perspective you would not see anything unusual. At this distance the tidal forces are near 1 million g. If you were free falling into the black hole (not slowing down your velocity), your velocity would be around 94% the speed of light.

And once you pass the Schwarzschild radius, you will be dead. At the rate you are moving that will happen in a tiny fraction of a second relative to your clocks (something like 0.000197 seconds). Your friends will never see you die, since you would appear frozen at the edge of the black hole (assuming they had some way of detecting the light from you).

Stephen Hawking

So nothing gets out of a black hole, right? Maybe. Stephen Hawking has come up with a way for something to get out of a black hole, or put another way, for a black hole to lose mass. This is via the emission of Hawking radiation. The situation requires that particles and anti-particles form near the Schwarzschild radius of the black hole. This creation of particles may take some energy from the black hole in the creation process. If one particle goes into the black hole and the other particle escapes, then as we (far from the black hole) measure the energy conservation, the particle that goes in actually has a negative energy – which would indicate a loss of mass for the black hole over time. So in this way the black hole has a specific temperature. Again, this is the realm of quantum physics where some things that seem impossible actually could happen.

The relation for the temperature produced by a black hole is given by

$$T = \frac{hc^3}{16\pi^2 GMk} \quad 8-9a$$

with the regular units of measure. But to make things easier, we can convert the constants to a value and measure the mass in solar masses, which gives us

$$T = \frac{6.2 \times 10^{-8}}{M} \quad 8-9b$$

where M is in solar masses, and the temperature is in Kelvin. Note that the temperature is inversely related to the mass – high mass, low temperature and the reverse, low mass, high temperature would also be the case. So a $1 M_{\odot}$ black hole would be very cold, while an even larger mass one would be even cooler. Only really small black holes are “hot”.

So if a black hole has a temperature and according to equation 8-1 it has a “radius”, that means we can define a luminosity for it – the rate at which it loses energy!

$$L = 4\pi R^2 \sigma T^4 = 4\pi \left(\frac{2GM}{c^2} \right)^2 \sigma \left(\frac{hc^3}{16\pi^2 GMk} \right)^4 = \frac{\Delta M c^2}{\Delta \tau} \quad 8-10$$

The last bit up there is sort of related to the Nuclear Time Scale for normal stars (equation 2-22), the rate at which stars give off energy and in the process lose mass. Here the mass is given by good old $E=mc^2$, and the time for mass/energy loss is $\Delta\tau$. This is basically the mass loss rate – not too different from that given for stellar winds. Okay it is really different, but it is still mass loss.

Fortunately this formula can be simplified since most of it is made up of constants and we get

$$\frac{\Delta M}{\Delta\tau} = \frac{\Delta Mass}{\Delta time} = \frac{1 \times 10^{-45}}{M^2} \quad 8-11$$

Where the value of M is in solar masses, and the mass loss rate is in M_{\odot} /second. So a $1 M_{\odot}$ black hole loses mass at the rate of $1 \times 10^{-45} M_{\odot}$ /second, or around 2×10^{-15} kg/second – which isn't really anything to get excited about.

Again, the more massive the black hole, the longer it will take to lose mass. The less massive ones will evaporate much more quickly.

You can actually calculate the mass of a black hole at any given time by doing a bit of calculus on the above formula. You would end up with

$$M(t) = (M_o^3 - 1.1 \times 10^{16} t)^{1/3} \quad 8-12a$$

Where M_o is the original mass (in kg) that you had and the time is in seconds.

The equivalent formula in units of solar masses is

$$M(t) = (M_o^3 - 1.4 \times 10^{-75} t)^{1/3} \quad 8-12b$$

You can use these formulae to determine the mass at any given time, t , including the time for it to evaporate completely. But that would be a very, very long time for most “normal” black holes.

Different types of black holes

So far we've only really mentioned the fact that black holes have mass and the boundaries that are involved with that. But if black holes come from stars, and stars have other characteristics, can't black holes also have some of those things as well?

Obviously for something to be a black hole, it must have mass. But what about rotation? Most stars rotate, some very quickly, some slowly. As a star collapses down, it would not stop rotating, but would likely spin faster, like the neutron stars speed up when they form in a core collapse. So if it collapses down to a black hole, shouldn't that also be spinning? And what about the magnetic field of the star? What about an electrical charge? While this is not as easy to deal with as the rotation, it is still a theoretical possibility. Theoreticians have looked at the possible black hole configurations and have come up with several possible types of black holes based upon the characteristics that they have.

Types of black holes -

- ⇒ Schwarzschild Black Hole – simplest, it has mass only.
- ⇒ Reissner –Nordstrom black hole – a bit more complex, has mass and charge.
- ⇒ Kerr black hole – this one has mass and rotation and is likely to exist.
- ⇒ Kerr-Newman black hole – all three things, but that’s sort of unlikely.

The ways to define these black holes are much more complex than just defining the good old Schwarzschild radius. We have to take into account the fact that space and time can be influenced by the black hole and depending upon the electrical charge and rotation, that influence can vary. If you go back to equation 8-7a or 8-7b, the idea of a metric is presented –this is a way of representing the relationship between distances, locations and time. With the curved space around a black hole you need these quantities. And since we are usually dealing with radial symmetry, it is easiest to use 8-7b.

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta (d\phi)^2 \quad 8-7b$$

A Kerr-Newman black hole is the most complex and it has quite an influence on space-time.

$$ds^2 = -\left(1 - \frac{2Mr - Q^2}{\Sigma}\right) (dt)^2 - \frac{(2Mr - Q^2)a \sin^2 \theta}{\Sigma} (dt)(d\phi) + \frac{\Sigma}{\Delta} (dr)^2 + \Sigma (d\theta)^2 + \frac{A \sin^2 \theta}{\Sigma} (d\phi)^2$$

$$\Delta = r^2 - 2Mr + a^2 + Q^2$$

$$\Sigma = r^2 + a^2 \cos^2 \theta$$

$$A = (r^2 + a^2)\Sigma + a^2(2Mr - Q^2) \sin^2 \theta$$

$$a = \frac{J}{M}$$

where Q =charge, M =Mass, and J =angular momentum. Actually this formula isn’t entirely “correct” since there are many values of c and G missing from the formula – ignoring those constants is generally okay since it is easier to just deal with values that are variable. Any ways, these relations give all of the ways that we define the influence of black holes to space-time. For the other types of black holes all you need to do is set $Q=0$ or $J=0$ and you’d get the corresponding metric for those black holes. Not a particularly fun thing, but that’s how it works.

While this relation looks icky, it does show a couple of limitations on black holes. One is a requirement for the black hole to actually exist if there is angular momentum or charge. This is (again without proper units) -

$$a^2 + Q^2 \leq M^2 \quad 8-13$$

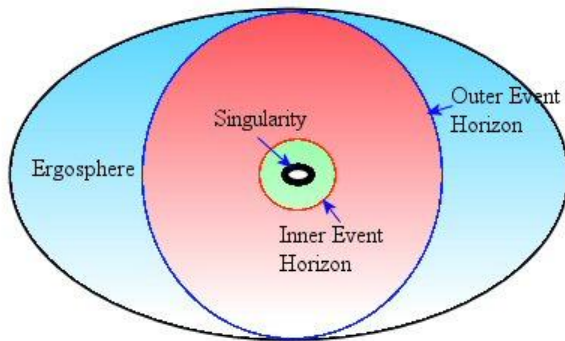
This puts a limit on the amount of charge and angular momentum that you can have for a black hole. But since it is unlikely for there to be charge (Q), it is really a limit on the angular momentum.

If you do have a rotating black hole (and odds are you will), you will have values for M and a and a side effect of the rotation – the dragging of space-time, also known as *frame dragging*. Space-time in the area of a massive object is altered by this effect, so someone

measuring something in orbit about a massive object should measure different passages of time due to the change of the space-time.

The spacecraft *Gravity Probe B* was launched to study the frame dragging near the earth. They used very precise gyroscopes to measure how much the earth's rotation alters the space around it. It also measured the amount of warping in the space around the earth (the geodetic effect). The amount of shift in the gyroscopes is predicted to be 6.606 arc-seconds due to the geodetic effect, and 0.039 arc-seconds due to frame dragging over the course of one year. So it isn't a really big effect but it is measurable. The preliminary results show that the geodetic effect was measured to an accuracy of 1%, but the data wasn't clean enough to get the frame dragging data out – after all, that is an effect that is much more difficult to see (170 times smaller than the geodetic effect).

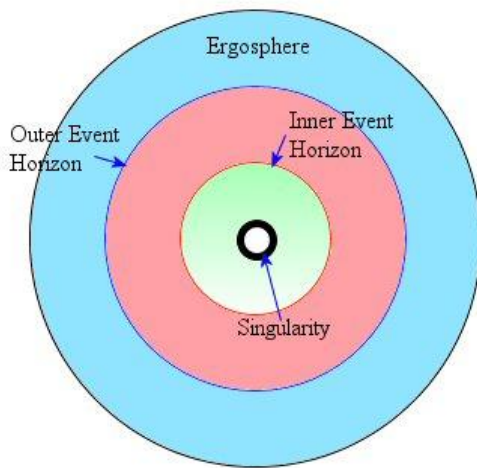
Let's look at spinning black holes a bit more. One peculiar aspect of a rotating black hole is that there are two event horizons for it rather than just one.



Here is a side view of such a black hole –

The ergosphere is the region around the black hole that gets dragged around by the rotation. You can't remain stationary there, you would be dragged along by the moving space-time. On the ergosurface (the outer boundary of the ergosphere), space-time is dragging around at the speed of light. Inside the ergosphere, space-

time is moving around at greater than the speed of light – this is legal, since it is SPACE-TIME, not a material substance, like you or your rocket. That's why you can't stand still, to do so would require velocities $>c$.



The outer event horizon touches the ergosphere at the poles – those are the axes of rotation for the black hole. Within the outer event horizon is another event horizon. And then the singularity is in the very center- however it is screwed up. The singularity is actually in a ring shape, not in a point! Below is a view of the black hole from above the axes of rotation – all of the boundaries are very symmetric from this perspective.

The rotation rates for the black holes have an influence on the sizes and relative locations for the event horizons. When the rotation rate increases, the inner event

horizon gets larger, while the outer event horizon gets smaller. There is a point when the two event horizons can merge together and cause the two event horizons to “vanish” – but this should not happen. This is basically like saying you have a black hole but no event horizon (boundary for escaping). This doesn’t make any sense because the black hole would be naked at that time – the so-called naked singularity. There is probably something that prevents a naked singularity from existing, but we haven’t figured that out yet.

Also looking back at equation 8-8 we have a limit on the rotation there. So there is a limit to how fast a black hole can rotate, it just can’t be spinning infinitely fast.

Looking at the black hole shown here, we see the ergosphere as the outer region. This is a region in which you cannot sit still, but would be pulled around quite fast. You can’t prevent this, the space would drag you. You can go towards or away from the black hole freely in this region, but you can’t go around it of your own accord.

Inside the ergosphere “time is space-like”. What does that mean? Time and space can be viewed as vectors. Time is supposed to go in one direction, always increasing steadily for you. Space has multiple directions relative to you. You can be stationary in space but not time.

In the ergosphere you can’t stand still – you are always moving. To stand still you would have to be moving at speeds greater than that of light, so that’s not possible. To an outside observer, the motion of the ergosphere gives the appearance of time flowing like space, so you can have the ability to change your time direction? Possibly – this would be to an outside observer, you always go forward in your own time.

It is also possible for you to have negative momentum or energy in this part of space – your motion could be such that going in a certain way you would add negative energy to a black hole. What good is this?

Let’s say you have an object with energy E_o . You break the object apart and send one part into the ergosphere with negative energy ($-E_e$). The other part comes away from the black hole. What would be the energy of this other part?

$$E_o = -E_e + E_{\text{final}}, \text{ so } E_{\text{final}} = E_o + E_e$$

So the energy of the object going away from the black hole is actually greater than the energy that we started with – we gained energy! This is a way of obtaining energy from a black hole (known as the Penrose process)!

How is it possible to have negative energy? One of the fun aspects of having space-time is that under certain conditions, space and time switch meaning – space becomes time-like and time becomes space-like. You can have negative space (down versus up) so when you allow time to do that as well, you can get negative time and therefore negatives of things you aren’t used to.

The only problem with extracting energy from a black hole in this manner is that the black hole will lose angular momentum in the process – it will slow down its spin. If you take too much energy from the black hole it will stop spinning.

As the rotation slows down the ergosphere will get smaller. That's not good, since you need it for the energy extraction process, so you can't do this too much.

One of the rules for black holes is the definition of the “surface area” –

$$A = \frac{8\pi G^2 M^2}{c^4} \left(1 + \sqrt{1 - \left(\frac{cJ}{GM^2} \right)^2} \right) \quad 8-14$$

This is for a rotating black hole (J=angular momentum). As $J \rightarrow 0$ you get the surface area for a Schwarzschild black hole.

This also puts the limits on the angular momentum since the radical can't become negative. So this means that angular momentum has values that range from 0 to GM^2/c .

One of the rules about black holes is that the surface area cannot decrease with time – it can increase, but it cannot decrease. This is because the surface area is related to the entropy of the black hole, and entropy only increases in a closed system (like the Universe). This has some other interesting repercussions.

Let's say you have two black holes, M_1 , and M_2 . And they have corresponding surface areas, A_1 , A_2 . You are going to smash them together and form a new black hole with mass M_3 , and surface area A_3 .

A_3 must be greater than $A_1 + A_2$.

If that is the case then (according to 84, with no rotation)

$$M_3^2 \geq M_1^2 + M_2^2$$

Note – this isn't an equality, so you can have a situation where $M_1 + M_2$ does not equal M_3 ! Also the square on the masses messes things up a bit in that respect.

So if $M_1 = 1$ solar mass, and $M_2 = 2$ solar masses then

$$1 + 4 = 5 = M_3^2$$

So $M_3 = 2.24$ solar masses or up to 3 solar masses at most.

The fact that you don't have to end up with 3 solar masses indicates that some mass can be lost in the black hole merger. And if mass is lost, that means some energy is also lost in the process. How much? Well just use good old $E = mc^2$ to determine the energy that comes off of it. The energy that is given off by the merger would be in the form of *gravitational radiation*. In electro-magnetism you can get EM radiation (light) from a charged accelerated particle. This is from ripples in EM fields due to the motion of charged particles. According to general relativity an accelerated mass should also produce gravitational radiation, by creating ripples in space-time (gravitational waves). Basically a very massive event can wiggle space time and this wiggling of space-time results in the creation of gravitational waves and the release of gravitational energy.

Obviously to get gravitational radiation you need to cause some major disruptions of space-time – events associated with high masses and a great deal of motion. Gravitational radiation may come from events such as supernovae, the accretion of large amounts of mass into a black hole, or a close binary system in a strongly distorted region of space-time.

Does this stuff actually occur?

In 1974, Russell Hulse and Joseph Taylor of Princeton University discovered a pulsar in a binary system, PSR 1913+16. This in itself isn't too unique, since other binary pulsar systems had been discovered, but this is the first case that the two stars were both neutron stars. When the stars are near one another, they are in a region of extreme gravitational fields, and this causes a distortion on the pulses from the pulsar. The time between the pulses lengthened in a manner that was predicted by the distortion of space due to general relativity effects. So there is a distortion to space due to binary motion on the pulses and a distortion due to the gravitational field of the objects. There is also a distortion on the orbits of each object since they are altering each other's orbits when they are close together. One orbit takes only about 8 hours. The orbits shift by a huge amount though, about 4.2 degrees per year. Orbit shifting is not something new, since we've seen Mercury's orbit shift due to the distortion of space near the Sun. But Mercury's orbit shifts by a measly 43"/century – much slower than the binary pulsar system!

The system is losing gravitational energy due to this interaction and with each orbit the orbital period is also changing. The rate of energy loss is in precise agreement with the predictions of general relativity. Eventually the stars will merge together and go BOOM. But that will probably not happen for about 300 million years.

Here are the parameters of the system –

Pulsar mass - $1.441 M_{\odot}$

Neutron star mass - $1.39 M_{\odot}$

Average distance apart – 1.9 million km (this is equivalent to about 1.5 solar diameters)

Orbital period – 7.751939106 hours

Pulsation period - .059 seconds

Location: Hercules, 6400 pc away.

PSR J0737-3039 is a binary pulsar system in which there are two pulsars. This was discovered in 2003 (too late for them to get the Nobel Prize, which Taylor and Hulse got for their pulsar). The orbital period of the system is 2.4 hours, which is much smaller than that of the previously described system. And as with PSR 1913+16, this one also matches all of the predications of Einstein. With the stars losing energy via gravitational waves, it is expected that these will coalesce in about 85 million years.

Here are the characteristics –

Pulsar masses – 1.24, and $1.35 M_{\odot}$

Pulsation periods – 2.8 seconds, and 0.023 seconds

Orbital period - 2.4 hours

Average distance apart – 800,000 km (a bit larger than the Sun’s radius)
Location: Canis Major, 2000 pc away

While these system do behave in the manner predicted with the loss of gravitational energy, we have yet to detect gravitational waves from anything. Those will be very difficult to detect, though there are several projects underway to find them. So stay tuned.

Possible Black Holes

Discussions about black holes also have to be taken with a grain of salt, since no one can “see” a black hole, and we can only infer its characteristics based upon the objects influence on its surroundings. But even without ever seeing a black hole, we do have a few very good candidates for black holes.

Cygnus X-1 is by far the most well known example of a likely black hole. This star is in the direction of the eta (η) star in the neck of Cygnus the swan. Way back in 1972 the x-ray source was discovered at this location, and an examination with a visible light telescope revealed a “normal” star HDE226868, which would not be the source for the x-rays. HDE226868 is a O9.7Iab star, which gives it a surface temperature that is around . Doppler shift measurements of the O star reveal that it is in orbit about the unseen x-ray source with a period of about 5.6 days. The orbit appears to be fairly circular, and the inclination angle is not precisely known, but may be close to 48° .

Observations indicate that the mass function, equation 1-16, would have a value of about

$$f(M_1, M_2) = 0.245 = \frac{M_x \sin^3 i}{(M_* + M_x)^2}$$

And if a value for the O star is assumed to be about 35 solar masses, then the solution for the above (assuming an inclination of 48°) is just over 10 solar masses for the x-ray mass, M_x . Different value for the O star or inclination angles will give different answers, but the result is undeniable that the x-ray source in Cygnus X-1 is very massive, yet is not a source for visible light.

V404 Cygni is a star that can’t decide what it wants to be. The name implies that it is a variable star in the constellation of Cygnus (wow, that’s a popular place for extreme stars), and historically it has been known to undergo a nova explosion. It also underwent an x-ray outburst, so it gets the classification of being an x-ray nova. Previous outbursts may have also emitted x-rays, but that was before we had x-ray telescopes. In this binary system the observed star is a cool K5 V, which orbits once every 6.5 days around the sporadically erupting x-ray source. The inclination of this object is fairly precisely known, between 52 and 60° , and with a mass function of 6.26, this gives a mass for the x-ray object somewhere between 10 - $15 M_\odot$. Some view this as a better candidate for a black hole because the characteristics of the orbit are defined more precisely. But the rock group Rush did a song for Cygnus X-1, and I don’t know if V404 Cygni is going to inspire any other musicians.

But both Cygnus X-1 and V404 Cygni are insignificant compared to IC 10 X-1. First of all, IC 10 is the name of a galaxy, which is about 825,000 pc away – this is part of our Local Group cluster of galaxies – in other words, it is nearby. IC 10 X-1 was discovered in this nearby galaxy, which has regions of excessive star formation. Due to the distance, it is a bit difficult to pin down the optical companion to the x-ray source, but the most likely object is a Wolf-Rayet star, 17A. Recent observations of this system, which has an orbital period of only 34.4 hours, which results in a mass function value of $7.64 \pm 1.26 M_{\odot}$. Now, Wolf-Rayet stars have rather imprecise values for mass, and the inclination of the orbit isn't well defined, but even using the widest possible values for the Wolf-Rayet star mass and inclination, the black hole candidates will still end up being somewhere between 20 and 40 M_{\odot} ! That's pretty impressive.

Of course one of the best known black hole candidates is not that with a stellar mass, but the object at the core of the galaxy, Sgr A*. This black hole is estimated to have a mass of about 2-3 million M_{\odot} . And Sgr A* is not unique – it appears that most galaxies have a massive black hole in their centers, which may indicate that black holes serve a very important role in the formation and/or evolution of galaxies. There is speculation that the earliest populations of stars in the universe may have formed these supermassive black holes around which galaxies formed, but that's fairly fuzzy at this point.

So there is quite a bit of evidence for million+ M_{\odot} black holes, and stellar mass black holes – what about something in between? Can there be a black hole with perhaps 1000 M_{\odot} or something that's 100,000 M_{\odot} ? So far there have only been a few possible candidates for intermediate mass black holes (IMBH), but nothing has held firm. It is possible that they don't exist, but just wait a few years, and someone will discover one....