Homework # 6

Name________________________
DUE: November 21 (at the start of class)

Your homework grade depends not only upon your getting the correct answer but also grammar, spelling and punctuation, particularly in questions that require explanations. Obviously numerical answers to problems do not need to be written in complete sentences. You will also be graded on the use of significant figures, proper units of measure and proper scientific notation. Partial credit may be given for showing your work even if your result is incorrect. You may work with others in determining the answers to the questions, but what you write should be in your own words – any homework assignments that look too similar to that of other students will receive no credit. Unless otherwise noted, all questions are worth 1 point.

1. (10 points total) **Radioactive Decay**. People often wonder how scientists are able to determine things such as the ages of rocks or the Earth. This is most frequently done by taking advantage of a neat process in nature called *radioactive decay*. When a radioactive substance breaks down it does so in a very systematic manner, and we can measure the rate of the break downs (or decays) to determine how old an object is. The rate of break down is often referred to as the *half-life*. For example, Uranium 238 (^{238}U) breaks down into Lead 206 (^{206}Pb) with a half-life of 4.5 billion years. This means that if you start out with a solid block of ^{238}U in 4.5 billion years 1/2 of the block will be ^{206}Pb. After each increment of the half-life, 1/2 of remaining radioactive material will decay, so with each step it is cut in half.

a. How much of a block that was originally all ^{238}U will still be ^{238}U and how much will be ^{206}Pb after 18 billion years?

b. (2 points) If you come across a block that is 3/4 ^{206}Pb and 1/4 ^{238}U, can you unambiguously determine it's age? If so, what is the age? If not, why not?

Now what are you supposed to do if you come across a block that has 72% ^{238}U and 28% ^{206}Pb, or a block that is 10.33 billion years old - how do you deal with these quantities to determine ages or amounts of material. Well, I'll tell you - you get to use some formulas (you knew those would pop up somewhere in here didn't you?) There are certain mathematical relations that help to determine ages and abundances and they make use of the number *e* - yes, that's a number. It's actually equal to something like 2.72. There is also a logarithmic function associated with this number (just like regular logarithms are associated with 10), but this one is called the *natural* *logarithm*. It is usually denoted on calculators by an Ln key, which is often the same as the e^x key. Now you'll finally get to use those keys!

First of all Ln works very much like Log does, so if you punch in Ln (24.5) you'll get an answer of 3.2. You can also take $e^{3.2}$ which gives an answer of 24.5. Pretty easy, isn't it?

c. What is the value of $e^{-2.94}$?

d. What is the value of Ln(15.4)?

The formula for calculating how much radioactive material is left after a given time is

$$P = e^{-\tau / r}$$

where P = the fraction (percentage) of the original radioactive material that still exists, τ = time in billions of years, r = decay rate (which is related to the 1/2 life value). Since P is always less than 1, you can make it a percentage of the material that remains (remember 0.35 = 35%). For ^{238}U the value of r is 6.5, so if you wanted to find out how much ^{238}U remains after a certain amount of time, the resulting formula would be

$$Percentage = e^{-\tau / 6.5}$$

e. So what percentage of a solid block of ^{238}U would remain after 6.69 billion years?

f. If you start with a solid block of ^{238}U, what would be the percentage that is still ^{238}U after four 1/2 lifes (= 18.0 billion years)?

Now going the other way, let's say you have a certain percentage of material that is still radioactive and you want to determine how old it is. In that case the formula would be

$$\tau = - r \, \mathrm{Ln}(P)$$

where the meaning of the terms is the same as before and r = 6.5 for the elements we are using here.

g. If you have a rock that is made up of 1.65% ^{238}U (and the rest is ^{206}Pb), how old is it (assuming it was all ^{238}U to begin with)?

h. What if you have a rock that is 85% ^{238}U - how old would it be (again assuming it was all originally ^{238}U)?

i. At what point (after how many years) would there be no ^{238}U left in a rock that started out made completely of ^{238}U?

2. (10 points total) Astronomers have discovered quite a few planets around other stars in our galaxy using Kepler's Third Law (that thing just keeps on popping up, doesn't it). The method of discovery is similar to that used for finding spectroscopic binary stars – the motion of the star gives away the presence of the massive planet around it.

Follow the link at the course website to find the velocity variation of a star due to the orbit of a planet about it. You'll see the velocity variation of the star over time. You can adjust the scale so that you can see at least two peaks or valleys. You'll also want to click on the "Best Fit" box – this puts the best fitting line through the data. Use this line to determine the orbital period for your star. When you click on the graph, the time for that location is displayed in units of days. Determine the time between successive peaks (or valleys), which will be the orbital period. Note: if you press "Get New Data" or reload the website, all of the data values will change – so you need to get all of your information at one time or you will have to start over.

a. What is the orbital period of the planet (which is the same as the period variation for the star) in days?

b. What is the orbital period as measured in years?

c. To determine the mass of the planet (m_p) you'll need to get the mass of the star – which means you need to make use of its spectral type. That information is also provided at the top of the graph. What is the luminosity for such a star (in terms of the Sun's luminosity)? (Hint: check those spectral characteristic tables from the second part of the course)

d. The luminosity of a Main Sequence star is related to its mass according to the relationship

$$L = M^{3.5}$$

where the luminosity and mass are given in terms of the Sun's values. What should the mass of your star be based upon this relation and the value you got in part c?

e. (2 points) Through various means we have determined that the orbit is 0.315 AU in size. If you go back to the notes about binary stars you would see that the mass of two objects in orbit about one another (in this case the star and the planet) can be found using Kepler's Third Law in the form

$$M_* + m_p = \frac{a^3}{P^2}$$

Where a=size of orbit in AU, P=period in years and M_* = mass of the star. The last thing to figure out of course is the mass of the planet (m_p). The masses of the star and planet are given in terms of the Sun's mass (solar masses). What is the value of m_p?

f. To convert this mass into kilograms you need to multiply it by the mass of the Sun, which is 2.00×10^{30} kg. How does the mass of this planet compare to the mass of Jupiter?

g. What is peculiar about the distance that this planet is from its star (at least peculiar when compared to our solar system)?

h. (2 points) Would you classify this planet as a terrestrial planet or a jovian planet? Make sure you give the reasons for your classification.