400:239 Advanced Statistics Homework 3 Answers (Fall 2009).

1. This calls for a direct-difference (repeated-measures) t-test. In SPSS you'd set the scores up in 2 columns corresponding to the groups:

	ShamPoo	Squeak
1	4.00	7.00
2	2.00	4.00
3	7.00	7.00
4	9.00	10.00
5	6.00	8.00
6	3.00	4.00
7	5.00	6.00
8	4.00	4.00
9	4.00	3.00
10	6.00	6.00
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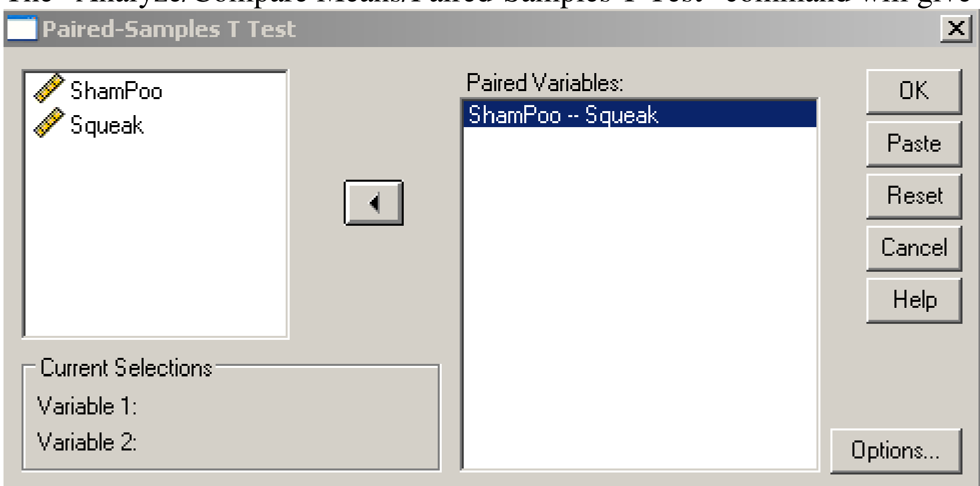
The "Analyze/Compare Means/Paired-Samples T Test" command will give you the correct t-test:



Above: the two variables selected

Output:

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	ShamPoo	5.0000	10	2.05480	.64979
	Squeak	5.9000	10	2.18327	.69041

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	ShamPoo & Squeak	10	.842	.002

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	95% Confidence Interval of the Difference: Upper			
Pair 1	ShamPoo - Squeak	-.90000	1.19722	.37859	-1.75644	-.04356	-2.377	9	.041

The significance level is .041, which is a smaller level than the alpha of .05, so Belfry should reject the null hypothesis that the two means are the same, and since the observed mean for Squeak was larger, the bats did appear to have significantly shinier coats when they were given Squeak shampoo.

2. a. We're told to do an independent groups t-test. In SPSS, you'd set this up coding a dummy variable representing the group the bat comes from:

	product	shininess
1	1.00	5.00
2	1.00	2.00
3	1.00	7.00
4	1.00	9.00
5	1.00	6.00
6	1.00	3.00
7	1.00	4.00
8	1.00	4.00
9	1.00	4.00
10	1.00	6.00
11	2.00	4.00
12	2.00	6.00
13	2.00	3.00
14	2.00	4.00
15	2.00	7.00
16	2.00	10.00
17	2.00	7.00
18	2.00	4.00
19	2.00	6.00
20	2.00	8.00
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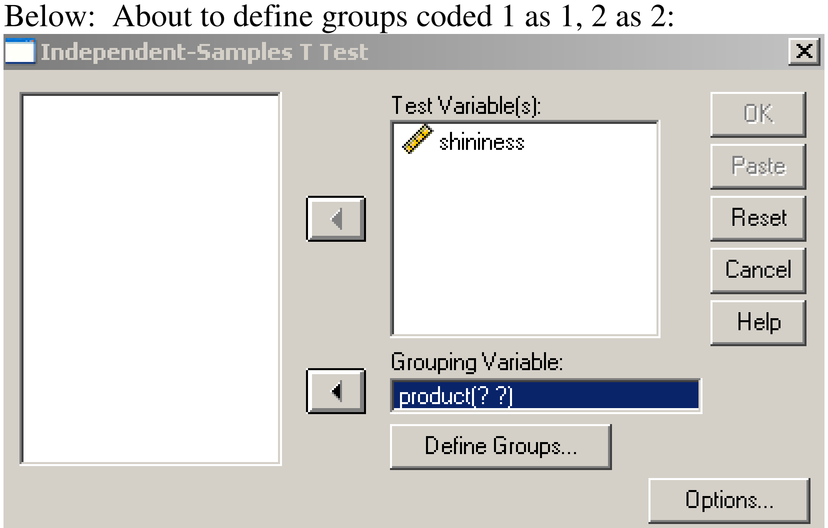
Then the "Analyze/Compare Means/Independent-Samples T Test" will perform both the pooled and separate variance versions of the t-test.

Below: About to define groups coded 1 as 1, 2 as 2:



Now the output:

Group Statistics

	product	N	Mean	Std. Deviation	Std. Error Mean
shininess	1.00	10	5.0000	2.05480	.64979
	2.00	10	5.9000	2.18327	.69041

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for E		
		F	Sig.	t	df	Sig. (2-tailed)
shininess	Equal variances assumed	.050	.825	-.949	18	.355
	Equal variances not assumed			-.949	17.934	.355

a. The pooled variance t is the upper line, showing an obtained t of -.949, with 18 df, and significance level of .355. That significance level falls above the alpha at .05, so here Belfry should fail to reject $H_0: \mu_1 = \mu_2$. That is, the difference does not appear to be significant.

b. The results reached contradictory conclusions, but were not unreasonable. The direct-difference test is more powerful, other things equal, and was able to detect what amounts to a smaller effect.

c. The Levene's test addresses the null hypothesis that $\sigma_1^2 = \sigma_2^2$; the significance level fell well above .05, so we wouldn't reject that null hypothesis--which in turn implies that it's reasonable to pool the variances, since they were similar.

Comments if done by hand:

You'd come to the same conclusions doing the calculations by hand. In both cases you could express the respective hypotheses as:
$H_C: \mu_1 < \mu_2$; $H_0: \mu_1 = \mu_2$ (which is equivalent to $H_0: \mu_2 - \mu_1 = 0$ and $H_0: \mu_D = 0$); and $H_1: \mu_1 \neq \mu_2$.
For Problem 1, *df* = *N* - 1 = 10 = 1 =9; for Problem 2, $df = n_1 + n_2 - 2 = 10 + 10 - 2 = 18$.
Problem 1:

$$\overline{X}_D = -.90;\ S_D = 1.197;\ S_{\overline{D}} = \frac{S_D}{\sqrt{N}} = \frac{1.197}{\sqrt{10}} = .379$$

$$t = \frac{\overline{X}_D - \mu_D}{S_{\overline{X}}} = \frac{-.90 - 0}{.379} = -2.38 \text{ and the critical } t_{.05}(9) = \pm 2.262 .$$

Problem 2:

$$\overline{X}_1 = 5.000\ \ S_1^2 = 4.223;\ \overline{X}_2 = 5.900\ \ S_2^2 = 4.765 .$$

$$S_P^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = 4.494$$

$$S_{\bar{X}_1 - \bar{X}_2} = \sqrt{S_P^2\left[\frac{1}{n_1} + \frac{1}{n_2}\right]} = .948$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_{\bar{X}_1 - \bar{X}_2}} = -.95$$

critical $t_{.05}(18) = \pm 2.101$

Alternatively to Levene's test in c, you could compute

$F_{\max} = \frac{S_L^2}{S_S^2} = \frac{4.765}{4.223} = 1.13$, which is less than 4 and would thus lead to the conclusion that the population variances appear similar, and therefore that the variances may be pooled.