**Elementary Logic Prof. Boedeker *Rules of inference* for propositional logic**

rule of inference Premise 1: / Premise 2: // Conclusion

modus ponens: (*a* ⊃ *c*) / *a* // *c*

modus tollens: (*a* ⊃ *c*) / ~*c* // ~*a*

pure hypothetical syllogism: (*a* ⊃ *b*) / (*b* ⊃ *c*) // (*a* ⊃ *c*)

disjunctive syllogism: (*a* ∨ *b*) / ~*a* // *b*

constructive dilemma: [(*a* ⊃ *b*) • (*c* ⊃ *d*)] / (*a* ∨ *c*) // (*b* ∨ *d*)

destructive dilemma: [(*a* ⊃ *b*) • (*c* ⊃ *d*)] / (~*b* ∨ ~*d*) // (~*a* ∨ ~*c*)

(These two “dilemmas” are less common forms of inference than the other ones above.)

The following pairs of statements are logically equivalent. Thus the first can be inferred from the second, and the second can be inferred from the first:

double negation: *a* ~~*a*

commutativity (for conjunction): (*a* • *b*) (*b* • *a*)

commutativity (for disjunction): (*a* ∨ *b*) (*b* ∨ *a*)

DeMorgan’s rule (form 1): ~(*a* • *b*) (~*a* ∨ ~*b*)

Not both *a* and *b*. Either not-*a* or not-*b*.

DeMorgan’s rule (form 2): ~(*a* ∨ *b*) (~*a* • ~*b*)

Neither *a* nor *b*. Both not-*a* and not-*b*.

**transposition: (*a* ⊃ *c*) (~*c* ⊃ ~*a*)**

**(very important!) If *a*, then *c*. If not-*c*, then not-*a*.**

**material implication: (*a* ⊃ *c*) (~*a* ∨ *c*)**

**(very important!) If *a*, then *c*. Not-*a* unless *c*.**

The rule of material implication exploits the fact that a conditional statement (which used to be called a “material implication”) is true if and only if either the antecedent “*a*”is false or the consequent “*c*”is true; and that’s just what “(~*a* ∨ *c*)” says.

material equivalence: (*a* ≡ *b*) [(*b* ⊃ *a*) • (*a* ⊃ *b*)] [(*a* ⊃ *b*) • (*b* ⊃ *a*)]

The rule of material equivalence exploits the fact that “(*a* ≡ *b*)” means “*a* if and only if *b*.” And this is a conjunction of two conditional statements: “Both *a* if *b*,and *a* only if *b*.” Now the first conditional statement, “*a* if *b*,” is expressed as “(*b* ⊃ *a*),” and the second conditional, “*a* only if *b*,” is expressed as “(*a* ⊃ *b*).” Thus “*a* if and only if *b*” means “Both *b* implies *a*,and *a* implies *b*,” which could – via “commutativity” – also be expressed as “Both *a* implies *b*,and *b* implies *a*”: “[(*a* ⊃ *b*) • (*b* ⊃ *a*)].”

Note that both rows below contain a set of logically-equivalent statements:

1. *a* unless *b*. (*a* ∨ *b*) (~*a* ⊃ *b*) (~*b* ⊃ *a*)

2. Not-*a* unless *b*. (~*a* ∨ *b*) (*a* ⊃ *b*) (~*b* ⊃ ~*a*)