

ON SENSE AND REFERENCE

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25] EQUALITY* gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my *Begriffsschrift*^A I assumed the latter. The reasons which seem to favour this are the following: $a=a$ and $a=b$ are obviously statements of differing cognitive value; $a=a$ holds *a priori* and, according to Kant, is to be labelled analytic, while statements of the form $a=b$ often contain very valuable extensions of our knowledge and cannot always be established *a priori*. The discovery that the rising sun is not new every morning, but always the same, was one of the most fertile astronomical discoveries. Even to-day the identification of a small planet or a comet is not always a
26] matter of course. Now if we were to regard equality as a relation between that which the names 'a' and 'b' designate, it would seem that $a=b$ could not differ from $a=a$ (i.e. provided $a=b$ is true). A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing. What is intended to be said by $a=b$ seems to be that the signs or names 'a' and 'b' designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted. But this relation would hold between the names or signs only in so far as they named or designated something. It would be mediated by the connexion of each of the two signs with

* I use this word in the sense of identity and understand 'a = b' to have the sense of 'a is the same as b' or 'a and b coincide.'

^A The reference is to Frege's *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (Halle, 1879).

the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrarily producible event or object as a sign for something. In that case the sentence $a=b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means. But in many cases this is just what we want to do. If the sign 'a' is distinguished from the sign 'b' only as object (here, by means of its shape), not as sign (i.e. not by the manner in which it designates something), the cognitive value of $a=a$ becomes essentially equal to that of $a=b$, provided $a=b$ is true. A difference can arise only if the difference between the signs corresponds to a difference in the mode of presentation of that which is designated. Let a, b, c be the lines connecting the vertices of a triangle with the midpoints of the opposite sides. The point of intersection of a and b is then the same as the point of intersection of b and c . So we have different designations for the same point, and these names ('point of intersection of a and b ,' 'point of intersection of b and c ') likewise indicate the mode of presentation; and hence the statement contains actual knowledge.

It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference of the sign, also what I should like to call the *sense* of the sign, wherein the mode of presentation is contained. In our example, accordingly, the
27] reference of the expressions 'the point of intersection of a and b ' and 'the point of intersection of b and c ' would be the same, but not their senses. The reference of 'evening star' would be the same as that of 'morning star,' but not the sense.

It is clear from the context that by 'sign' and 'name' I have here understood any designation representing a proper name, which thus has as its reference a definite object (this word taken in the widest range), but not a concept or a relation, which shall be discussed further in another article.^B The designation of a single object can also consist of several words or other signs. For brevity, let every such designation be called a proper name.

The sense of a proper name is grasped by everybody who is sufficiently familiar with the language or totality of designations

^B See his 'Ueber Begriff und Gegenstand' (*Vierteljahrsschrift für wissenschaftliche Philosophie* XVI [1892], 192-205).

to which it belongs;* but this serves to illuminate only a single aspect of the reference, supposing it to have one. Comprehensive knowledge of the reference would require us to be able to say immediately whether any given sense belongs to it. To such knowledge we never attain.

The regular connexion between a sign, its sense, and its reference is of such a kind that to the sign there corresponds a definite sense and to that in turn a definite reference, while to a given reference (an object) there does not belong only a single sign. The same sense has different expressions in different languages or even in the same language. To be sure, exceptions to this regular behaviour occur. To every expression belonging to a complete totality of signs, there should certainly correspond 28] a definite sense; but natural languages often do not satisfy this condition, and one must be content if the same word has the same sense in the same context. It may perhaps be granted that every grammatically well-formed expression representing a proper name always has a sense. But this is not to say that to the sense there also corresponds a reference. The words 'the celestial body most distant from the Earth' have a sense, but it is very doubtful if they also have a reference. The expression 'the least rapidly convergent series' has a sense but demonstrably has no reference, since for every given convergent series, another convergent, but less rapidly convergent, series can be found. In grasping a sense, one is not certainly assured of a reference.

If words are used in the ordinary way, what one intends to speak of is their reference. It can also happen, however, that one wishes to talk about the words themselves or their sense. This happens, for instance, when the words of another are quoted. One's own words then first designate words of the other speaker, and only the latter have their usual reference. We then have signs of signs. In writing, the words are in this case enclosed in quotation marks. Accordingly, a word standing between

* In the case of an actual proper name such as 'Aristotle' opinions as to the sense may differ. It might, for instance, be taken to be the following: the pupil of Plato and teacher of Alexander the Great. Anybody who does this will attach another sense to the sentence 'Aristotle was born in Stagira' than will a man who takes as the sense of the name: the teacher of Alexander the Great who was born in Stagira. So long as the reference remains the same, such variations of sense may be tolerated, although they are to be avoided in the theoretical structure of a demonstrative science and ought not to occur in a perfect language.

quotation marks must not be taken as having its ordinary reference.

In order to speak of the sense of an expression 'A' one may simply use the phrase 'the sense of the expression "A"'. In reported speech one talks about the sense, e.g., of another person's remarks. It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. In order to have a short expression, we will say: In reported speech, words are used *indirectly* or have their *indirect* reference. We distinguish accordingly the *customary* from the *indirect* reference of a word; and its *customary* sense from its *indirect* sense. The indirect reference of a word is accordingly its customary sense. Such exceptions must always be borne in mind if the mode of connexion between sign, sense, and reference in particular cases is to be correctly understood.

29] The reference and sense of a sign are to be distinguished from the associated idea. If the reference of a sign is an object perceivable by the senses, my idea of it is an internal image,* arising from memories of sense impressions which I have had and acts, both internal and external, which I have performed. Such an idea is often saturated with feeling; the clarity of its separate parts varies and oscillates. The same sense is not always connected, even in the same man, with the same idea. The idea is subjective: one man's idea is not that of another. There result, as a matter of course, a variety of differences in the ideas associated with the same sense. A painter, a horseman, and a zoologist will probably connect different ideas with the name 'Bucephalus.' This constitutes an essential distinction between the idea and the sign's sense, which may be the common property of many and therefore is not a part or a mode of the individual mind. For one can hardly deny that mankind has a common store of thoughts which is transmitted from one generation to another.†

* We can include with ideas the direct experiences in which sense-impressions and acts themselves take the place of the traces which they have left in the mind. The distinction is unimportant for our purpose, especially since memories of sense-impressions and acts always go along with such impressions and acts themselves to complete the perceptual image. One may on the other hand understand direct experience as including any object, in so far as it is sensibly perceptible or spatial.

† Hence it is inadvisable to use the word 'idea' to designate something so basically different.

In the light of this, one need have no scruples in speaking simply of *the* sense, whereas in the case of an idea one must, strictly speaking, add to whom it belongs and at what time. It might perhaps be said: Just as one man connects this idea, and another that idea, with the same word, so also one man can associate this sense and another that sense. But there still remains a difference in the mode of connexion. They are not prevented from grasping the same sense; but they cannot have the same 30] idea. *Si duo idem faciunt, non est idem*. If two persons picture the same thing, each still has his own idea. It is indeed sometimes possible to establish differences in the ideas, or even in the sensations, of different men; but an exact comparison is not possible, because we cannot have both ideas together in the same consciousness.

The reference of a proper name is the object itself which we designate by its means; the idea, which we have in that case, is wholly subjective; in between lies the sense, which is indeed no longer subjective like the idea, but is yet not the object itself. The following analogy will perhaps clarify these relationships. Somebody observes the Moon through a telescope. I compare the Moon itself to the reference; it is the object of the observation, mediated by the real image projected by the object glass in the interior of the telescope, and by the retinal image of the observer. The former I compare to the sense, the latter is like the idea or experience. The optical image in the telescope is indeed one-sided and dependent upon the standpoint of observation; but it is still objective, inasmuch as it can be used by several observers. At any rate it could be arranged for several to use it simultaneously. But each one would have his own retinal image. On account of the diverse shapes of the observers' eyes, even a geometrical congruence could hardly be achieved, and an actual coincidence would be out of the question. This analogy might be developed still further, by assuming A's retinal image made visible to B; or A might also see his own retinal image in a mirror. In this way we might perhaps show how an idea can itself be taken as an object, but as such is not for the observer what it directly is for the person having the idea. But to pursue this would take us too far afield.

We can now recognize three levels of difference between words,

expressions, or whole sentences. The difference may concern at most the ideas, or the sense but not the reference, or, finally, the reference as well. With respect to the first level, it is to be 31] noted that, on account of the uncertain connexion of ideas with words, a difference may hold for one person, which another does not find. The difference between a translation and the original text should properly not overstep the first level. To the possible differences here belong also the colouring and shading which poetic eloquence seeks to give to the sense. Such colouring and shading are not objective, and must be evoked by each hearer or reader according to the hints of the poet or the speaker. Without some affinity in human ideas art would certainly be impossible; but it can never be exactly determined how far the intentions of the poet are realized.

In what follows there will be no further discussion of ideas and experiences; they have been mentioned here only to ensure that the idea aroused in the hearer by a word shall not be confused with its sense or its reference.

To make short and exact expressions possible, let the following phraseology be established:

A proper name (word, sign, sign combination, expression) *expresses* its sense, *stands for* or *designates* its reference. By means of a sign we express its sense and designate its reference.

Idealists or sceptics will perhaps long since have objected: 'You talk, without further ado, of the Moon as an object; but how do you know that the name 'the Moon' has any reference? How do you know that anything whatsoever has a reference?' I reply that when we say 'the Moon,' we do not intend to speak of our idea of the Moon, nor are we satisfied with the sense alone, but we presuppose a reference. To assume that in the sentence 'The Moon is smaller than the Earth' the idea of the Moon is in question, would be flatly to misunderstand the sense. If this is what the speaker wanted, he would use the phrase 'my idea of the Moon.' Now we can of course be mistaken in the presupposition, and such mistakes have indeed occurred. But the question whether the presupposition is perhaps always mistaken 32] need not be answered here; in order to justify mention of the reference of a sign it is enough, at first, to point out our intention

in speaking or thinking. (We must then add the reservation: provided such reference exists.)

So far we have considered the sense and reference only of such expressions, words, or signs as we have called proper names. We now inquire concerning the sense and reference for an entire declarative sentence. Such a sentence contains a thought.* Is this thought, now, to be regarded as its sense or its reference? Let us assume for the time being that the sentence has reference. If we now replace one word of the sentence by another having the same reference, but a different sense, this can have no bearing upon the reference of the sentence. Yet we can see that in such a case the thought changes; since, e.g., the thought in the sentence 'The morning star is a body illuminated by the Sun' differs from that in the sentence 'The evening star is a body illuminated by the Sun.' Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false. The thought, accordingly, cannot be the reference of the sentence, but must rather be considered as the sense. What is the position now with regard to the reference? Have we a right even to inquire about it? Is it possible that a sentence as a whole has only a sense, but no reference? At any rate, one might expect that such sentences occur, just as there are parts of sentences having sense but no reference. And sentences which contain proper names without reference will be of this kind. The sentence 'Odysseus was set ashore at Ithaca while sound asleep' obviously has a sense. But since it is doubtful whether the name 'Odysseus,' occurring therein, has reference, it is also doubtful whether the whole sentence has one. Yet it is certain, nevertheless, that anyone who seriously took the sentence to be true or false would ascribe to the name 'Odysseus' a reference, not merely 33] a sense; for it is of the reference of the name that the predicate is affirmed or denied. Whoever does not admit the name has reference can neither apply nor withhold the predicate. But in that case it would be superfluous to advance to the reference of the name; one could be satisfied with the sense, if one wanted to go no further than the thought. If it were a question only of the sense of the sentence, the thought, it would be

* By a thought I understand not the subjective performance of thinking but its objective content, which is capable of being the common property of several thinkers.

unnecessary to bother with the reference of a part of the sentence; only the sense, not the reference, of the part is relevant to the sense of the whole sentence. The thought remains the same whether 'Odysseus' has reference or not. The fact that we concern ourselves at all about the reference of a part of the sentence indicates that we generally recognize and expect a reference for the sentence itself. The thought loses value for us as soon as we recognize that the reference of one of its parts is missing. We are therefore justified in not being satisfied with the sense of a sentence, and in inquiring also as to its reference. But now why do we want every proper name to have not only a sense, but also a reference? Why is the thought not enough for us? Because, and to the extent that, we are concerned with its truth value. This is not always the case. In hearing an epic poem, for instance, apart from the euphony of the language we are interested only in the sense of the sentences and the images and feelings thereby aroused. The question of truth would cause us to abandon aesthetic delight for an attitude of scientific investigation. Hence it is a matter of no concern to us whether the name 'Odysseus,' for instance, has reference, so long as we accept the poem as a work of art.* It is the striving for truth that drives us always to advance from the sense to the reference.

We have seen that the reference of a sentence may always be sought, whenever the reference of its components is involved; and that this is the case when and only when we are inquiring after the truth value.

34] We are therefore driven into accepting the *truth value* of a sentence as constituting its reference. By the truth value of a sentence I understand the circumstance that it is true or false. There are no further truth values. For brevity I call the one the True, the other the False. Every declarative sentence concerned with the reference of its words is therefore to be regarded as a proper name, and its reference, if it has one, is either the True or the False. These two objects are recognized, if only implicitly, by everybody who judges something to be true—and so even by a sceptic. The designation of the truth values as objects may

* It would be desirable to have a special term for signs having only sense. If we name them, say, representations, the words of the actors on the stage would be representations; indeed the actor himself would be a representation.

appear to be an arbitrary fancy or perhaps a mere play upon words, from which no profound consequences could be drawn. What I mean by an object can be more exactly discussed only in connexion with concept and relation. I will reserve this for another article.^c But so much should already be clear, that in every judgment,* no matter how trivial, the step from the level of thoughts to the level of reference (the objective) has already been taken.

One might be tempted to regard the relation of the thought to the True not as that of sense to reference, but rather as that of subject to predicate. One can, indeed, say: 'The thought, that 5 is a prime number, is true.' But closer examination shows that nothing more has been said than in the simple sentence '5 is a prime number.' The truth claim arises in each case from the form of the declarative sentence, and when the latter lacks its usual force, e.g., in the mouth of an actor upon the stage, even the sentence 'The thought that 5 is a prime number is true' contains only a thought, and indeed the same thought as the simple '5 is a prime number.' It follows that the relation of the thought to the True may not be compared with that of subject to predicate. 35] Subject and predicate (understood in the logical sense) are indeed elements of thought; they stand on the same level for knowledge. By combining subject and predicate, one reaches only a thought, never passes from sense to reference, never from a thought to its truth value. One moves at the same level but never advances from one level to the next. A truth value cannot be a part of a thought, any more than, say, the Sun can, for it is not a sense but an object.

If our supposition that the reference of a sentence is its truth value is correct, the latter must remain unchanged when a part of the sentence is replaced by an expression having the same reference. And this is in fact the case. Leibniz gives the definition: '*Eadem sunt, quae sibi mutuo substitui possunt, salva veritate.*' What else but the truth value could be found, that belongs quite generally to every sentence if the reference of its components is relevant, and remains unchanged by substitutions of the kind in question?

^c See his 'Ueber Begriff und Gegenstand' (*Vierteljahrsschrift für wissenschaftliche Philosophie* XVI [1892], 192-205).

* A judgment, for me is not the mere comprehension of a thought, but the admission of its truth.

If now the truth value of a sentence is its reference, then on the one hand all true sentences have the same reference and so, on the other hand, do all false sentences. From this we see that in the reference of the sentence all that is specific is obliterated. We can never be concerned only with the reference of a sentence; but again the mere thought alone yields no knowledge, but only the thought together with its reference, i.e. its truth value. Judgments can be regarded as advances from a thought to a truth value. Naturally this cannot be a definition. Judgment is something quite peculiar and incomparable. One might also say that judgments are distinctions of parts within truth values. Such distinction occurs by a return to the thought. To every sense belonging to a truth value there would correspond its own manner of analysis. However, I have here used the word 'part' in a special sense. I have in fact transferred the relation between the parts and the whole of the sentence to its reference, by calling the reference of a word part of the reference of the sentence, if the 36] word itself is a part of the sentence. This way of speaking can certainly be attacked, because the whole reference and one part of it do not suffice to determine the remainder, and because the word 'part' is already used in another sense of bodies. A special term would need to be invented.

The supposition that the truth value of a sentence is its reference shall now be put to further test. We have found that the truth value of a sentence remains unchanged when an expression is replaced by another having the same reference: but we have not yet considered the case in which the expression to be replaced is itself a sentence. Now if our view is correct, the truth value of a sentence containing another as part must remain unchanged when the part is replaced by another sentence having the same truth value. Exceptions are to be expected when the whole sentence or its part is direct or indirect quotation; for in such cases, as we have seen, the words do not have their customary reference. In direct quotation, a sentence designates another sentence, and in indirect quotation a thought.

We are thus led to consider subordinate sentences or clauses. These occur as parts of a sentence complex, which is, from the logical standpoint, likewise a sentence—a main sentence. But here we meet the question whether it is also true of the