**Elementary Logic**

**Handout on using predicate logic to understand categorical logic**

Our textbook gives us a number of rules to memorize to determine whether various pairs of categorical propositions are logically equivalent or not. These rules are summarized in the table at the end of section 4.4. Of course, you’re free to memorize the rules if you like, but it’s almost always better to really understand material than simply to memorize facts about it. For one thing, what we understand tends to stick with us longer than what we merely memorize. And, fortunately, our study of predicate logic and Venn diagrams allows us to forgo this memorization. Furthermore, we’re now in a position to determine much more than what’s summarized in the table. For we can actually determine not just whether two categorical propositions are logically equivalent, but also whether they’re mutually contradictory, or consistent but not logically equivalent. Understanding how to manipulate categorical propositions as indicated in this handout will also prove very useful as we move on to Chapter 5, where we’ll be using Venn diagrams to prove arguments valid or invalid in categorical logic.

This technique involves an understanding of three things:

First, we need to remember how to express categorical propositions in predicate logic, and vice-versa. How to do this is indicated in the diagram on the other handout for this assignment.

Second, we need to remember how to apply two (or, technically, three) operations *within* a proposition expressed in predicate logic:

double negation:

~~*a* = *a*

transposition:

(*a* ⊃ *b*) = (~*b* ⊃ ~*a*)

commutativity (used very infrequently, and usually just for particular statements):

(*a* ⋅ *b*) = (*b* ⋅ *a*)

Whenever we apply any of these operations to a statement, we get a logically equivalent statement.

Third, we need to remember how to express statements in categorical logic in Venn diagrams.

Ultimately, there are exactly eight kinds of categorical propositions, as indicated in the diagram on the other handout for this assignment:

(1) A universal affirmative “All *S* are *P*” is mutually contradictory with (2) its corresponding particular negative: “Some *S* are not *P*.”

(3) A universal negative “No *S* are *P*” is mutually contradictory with (4) its corresponding particular affirmative: “Some *S* are *P*.”

(5) A converse universal affirmative, “All *P* are *S*,” is mutually contradictory with (6) its corresponding converse particular negative: “Some *P* are not *S*.”

(7) A contrapositive universal negative, “All non-*S* are *P*,” is mutually contradictory with (8) its corresponding contrapositive particular negative: “Some non-*S* are non-*P*.”

No other immediate logical relations hold among any other pairs of propositions. That is, every other pair is consistent.

Our technique consists in following these steps:

**Step 1:** Express the original propositions in categorical logic.

If any of the original propositions is not yet expressed in one of the eight standard forms (as indicated in the diagram in the other handout for this homework assignment), move on to **step 2**. If they are already expressed in one of these eight standard forms, skip ahead to **step 5**.

**Step 2:** Express the categorical propositions in predicate logic.

If any of the propositions expressed in *predicate logic* is not yet expressed in one of the eight standard forms (as indicated in the diagram in the other handout for this homework assignment), move on to **step 3**. If they are already expressed in one of these eight standard forms, skip ahead to **step 4**.

**Step 3:** Apply the operations of double negation or transposition (and sometimes, technically, commutativity) within the quantified propositions in order to express the proposition (in predicate logic) in one of our eight standard forms.

**Step 4:** Write out the resulting propositions in categorical logic (“Some *S* are *P*”, “No *S* are *P*”, “All non-*S* are *P*,” etc.).

**Step 5:** Draw the Venn diagram for the propositions, corresponding to their expressions in standard form.

Following these steps will allow us to see in a vivid way just whether any pair of categorical propositions is logically equivalent, mutually contradictory, or consistent but not logically equivalent.

Here are a number of examples of how to do this, taken from the exercises in section 4.4. Obviously, in what follows I’m giving a lot of examples, so just read as much of the explanations as you find helpful. *Be sure, however, to read the discussion of* ***commutativity*** *at the end of this document.*

4.4: I, exercise 1:

**Original statement:** Step 1: “No *A* are non-*B*” is true.

Step 2: This is expressed in predicate logic as “(*x*)(*Ax* ⊃ ~~*Bx*)”.

Step 3: Applying double negation to “~~*Bx*”) gives us “(*x*)(*Ax* ⊃ *Bx*)”.

Step 4: This last expression is our standard way to express in predicate logic a universal affirmative, so we express it in categorical logic as “All *A* are *B*.”

Step 5: Since “All *A* are *B*” is a universal affirmative, it’s expressed in a Venn diagram by shading the area in *A* outside of *B*.

**New statement:** Step 1: “No non-*B* are *A*.”

Step 2: This is expressed in predicate logic as “(*x*)(~*Bx* ⊃ ~*Ax*)”.

Step 3: Applying transposition to “(~*Bx* ⊃ ~*Ax*) gives us “(*x*)(*Ax* ⊃ *Bx*)”.

Step 4: This last expression is our standard way to express in predicate logic a universal affirmative, so we express it in categorical logic as “All *A* are *B*.”

Step 5: Since this is the very same proposition as our original, it naturally is expressed using the same Venn diagram, and the two propositions are logically equivalent. Hence, because we’re told that the original statement is true, the new statement is also *true*.

4.4: I, exercise 4:

**Original statement:** Step 1: “All non-*A* are *B*” is false.

Step 2: This is expressed in predicate logic as follows: “(*x*)(~*Ax* ⊃ *Bx*)” is false.

This last expression is already the standard way to express in predicate logic one of the eight forms of categorical statements; it’s the claim that a contrapositive universal negative is false. Thus we skip ahead to step (4).

Step 4: Expressed in categorical logic, this is the claim that “All non-*A* are *B*” is false.

Step 5: Note (as shown on our diagram) that this proposition is false if and only if its corresponding contrapositive particular affirmative, “Some non-*A* are non-*B*,” is true. And this is expressed in a Venn diagram by placing an X in the area outside of both *A* and *B*.

**New statement:** Step 1: “All non-*B* are *A*.”

Step 2: This is expressed in predicate logic as “(*x*)(~*Bx* ⊃ *Ax*)”.

This is already the standard way to express in predicate logic a categorical proposition in standard form: a contrapositive universal negative. Thus we skip ahead to step (4).

Step 4: We express this contrapositive universal negative in categorical logic as “All non-*B* are *A*.”

Step 5: We express this contrapositive universal negative in a Venn diagram by shading the area outside of both *A* and *B*.

When we compare the two Venn diagrams, we see that the falsehood of the original statement implies that there is something outside of both *A* and *B*, whereas our new statement implies that there’s nothing in this region. Because we’re told that the original statement is false, the new statement is *false* as well.

4.4: I, exercise 7:

**Original statement:** Step 1: “No non-*A* are non-*B*” is false.

Step 2: This is expressed in predicate logic as follows: “(*x*)(~*Ax* ⊃ ~~*Bx*)” is false.

Step 3: Applying double negation to “~~*Bx*” gives us the following: “(*x*)(~*Ax* ⊃ *Bx*)” is false.

Step 4: The last expression is the standard way to express in predicate logic the claim that the contrapositive universal negative “All non-*A* are *B*” is false.

Step 5: Note (as shown on the diagram) that this contrapositive universal negative is false if and only if its corresponding contrapositive particular affirmative, “Some non-*A* are non-*B*,” is true. And this is expressed in a Venn diagram by placing an X in the area outside of both *A* and *B*.

**New statement:** Step 1: “No *B* are *A*.”

This statement is already in the standard form: it’s a universal negative. Thus we skip ahead to step (5).

Step 5: Our new statement is expressed in a Venn diagram by shading the overlap between *A* and *B*.

Comparing the Venn diagrams for our two propositions reveals that, although the new statement is *consistent* with the falsehood of our given statement, this new statement is neither logically equivalent nor mutually contradictory to it. The truth-value of the new statement is thus *undetermined*.

4.4: I, exercise 10:

**Original statement:** Step 1: “No non-*A* are *B*” is false.

Step 2: This is expressed in predicate logic as follows: “(*x*)(~*Ax* ⊃ ~*Bx*)” is false.

Step 3: Applying transposition to “(~*Ax* ⊃ ~*Bx*)” gives us the following: “(*x*)(*Bx* ⊃ *Ax*)” is false.

Step 4: “(*x*)(*Bx* ⊃ *Ax*)” is our standard way to express in predicate logic the universal affirmative “All *B* are *A*”, which we’re told is false.

Step 5: As indicated in our diagram, a universal affirmative – in this case, “All *B* are *A*” – is false if and only if its corresponding particular affirmative – in this case, “Some *B* are not *A*” – is true. Hence, since our universal affirmative is expressed by shading the area in *B* outside of *A*, we express the falsehood of the original statement by placing an X in the area in *B* outside of *A*.

**New statement:** Step 1: “All non-*A* are non-*B*”.

Step 2: This is expressed in predicate logic as follows: “(*x*)(~*Ax* ⊃ ~*Bx*)”.

Step 3: Applying transposition to “(~*Ax* ⊃ ~*Bx*)” gives us the following: “(*x*)(*Bx* ⊃ *Ax*).”

Step 4: This is our standard way to express in predicate logic the universal affirmative “All *B* are *A*.” And this is logically equivalent to the original statement, which we’re told is false.

We don’t even need to construct a Venn diagram to determine the truth-value of our new statement. Since the original statement and the new statement are logically equivalent, and we’re told that the original statement is false, the new statement is also *false*.

Now let’s move on to some exercises in 4.4: III:

4.4: III, exercise 1:

First, we give our scheme of abbreviation for our two terms:

*C*: commodity traders.

*G*: gamblers who risk sudden disaster.

Our corresponding predicates are:

*Cx*: *x* is a commodity trader.

*Gx*: *x* is a gambler who risks sudden disaster.

**Premise** (step 1)**: All *C* are *G*.**

This is already the standard way to express in categorical logic a universal affirmative, so we skip ahead to step (5).

Step 5: We express this universal affirmative in a Venn diagram by shading the area in *C* outside of *G*.

**Conclusion** (step 1)**: All *G* are *C*.**

This is already the standard way to express in categorical logic a universal affirmative, so we skip ahead to step (5).

Step 5: We express the universal affirmative “All *G* are *C*” in a Venn diagram by shading the area in *G* outside of *C*.

When we compare the Venn diagram of the premise with the Venn diagram of the conclusion, we see that the conclusion is the converse of the premise, since the opposite region is shaded. Thus although these statements are consistent, the premise doesn’t imply the conclusion. Hence the argument is *invalid*.

4.4: III, exercise 4:

First, we give our scheme of abbreviation for our two *positive* terms:

*S*: sane people

*L*: logical people.

Our corresponding predicates are:

*Sx*: *x* is a sane person.

*Lx*: *x* is a logical person.

**Premise** (step 1)**: Some non-*S* are non-*L*.**

This is already the standard way to express in categorical logic a contrapositive particular affirmative, so we skip ahead to step (5).

Step 5: This contrapositive particular affirmative is expressed in a Venn diagram by placing an X outside of both *S* and *L*.

**Conclusion** (step 1)**: Some *L* are *S*.**

This is already the standard way to express in categorical logic a particular affirmative, so we skip ahead to step (5).

Step 5: This particular affirmative is expressed in a Venn diagram by placing an X in the overlap between *S* and *L*.

When we compare the Venn diagrams of the two statements, we see that, although the conclusion is *consistent* with the premise, it’s not entailed by the premise. Hence the argument is *invalid*.

4.4: III, exercise 7:

First, we give our scheme of abbreviation for the two *positive* terms:

*H*: periods when interest rates are high.

*E*: periods when businesses tend to expand.

Our corresponding predicates are:

*Hx*: *x* is a period when interest rates are high.

*Ex*: *x* is a period when businesses tend to expand.

**Premise** (step 1)**: All *H* are non-*E*.**

Step 2: This is expressed in predicate logic as “(*x*)(*Hx* ⊃ ~*Ex*)”.

This last expression is already our standard way to express in predicate logic a universal negative, so we skip ahead to step (4).

Step 4: We express this statement in categorical logic as “No *H* are *E*.”

Step 5: This universal negative is expressed in a Venn diagram by shading the overlap between *H* and *E*.

Next, to express the **conclusion**, note that “times when interest rates are low” is the *term complem*ent of “times when interest rates are high.” We thus express the former term as “non-*H*”. Hence our conclusion is the following:

**Conclusion** (step 1)**: All *E* are non-*H*.**

Step 2: This is expressed in predicate logic as “(*x*)(*Ex* ⊃ ~*Hx*)”.

This last expression is already is our standard way to express in predicate logic a universal negative, so we skip ahead to step (4).

Step 4: We express this statement in categorical logic as “No *E* are *H*.”

Step 5: This universal negative is expressed in a Venn diagram by shading the overlap between *E* and *H*.

The Venn diagram expressing the conclusion is exactly the same as the Venn diagram expressing the premise. The conclusion is thus logically equivalent to the premise, and the argument is therefore *valid*.

4.4: III, exercise 10:

First, we give our scheme of abbreviation for the two positive terms:

*N*: ladies of the night.

*L*: individuals with low self-esteem.

Our corresponding predicates are:

*Nx*: *x* is a lady of the night.

*Lx*: *x* is an individual with low self-esteem.

**Premise** (step 1)**: All *N* are *L*.**

This is the standard way to express in categorical logic a universal affirmative, so we skip ahead to step (5).

Step 5: Our universal affirmative is expressed in a Venn diagram by shading the area in *N* outside of *L*.

Next, to express the **conclusion**, note that “individuals with high self-esteem” is the *term complement* of “individuals with low self-esteem.” We thus express the former term as “non-*L*”. Hence our conclusion is the following:

**Conclusion** (step 1): **No *N* are non-*L*.**

Step 2: This is expressed in predicate logic as “(*x*)(*Nx* ⊃ ~~*Lx*)”.

Step 3: Applying double negation to “~~*Lx*” gives us “(*x*)(*Nx* ⊃ *Lx*)”.

Step 4: This is our standard way to express in predicate logic the universal affirmative “All *N* are *L*.”

We don’t even need to construct a Venn diagram for the conclusion to determine whether the argument is valid. Since the premise and the conclusion are logically equivalent, the argument is *valid*.

4.4: III, exercise 13:

First, we give our scheme of abbreviation for the two terms:

*I*: insurance companies.

*H*: humanitarian organizations.

Our corresponding predicates are:

*Ix*: *x* is an insurance company.

*Hx*: *x* is a humanitarian organization.

**Premise** (step 1)**: Some *I* are not *H*.**

This is the standard way to express in categorical logic a particular negative, so we skip ahead to step (5).

Step 5: Our particular negative is expressed in a Venn diagram by placing an X in the area in *I* outside of *H*.

**Conclusion** (step 1): **Some *H* are not *I*.**

This is the standard way to express in categorical logic a particular negative, so we skip ahead to step (5).

Step 5: Our particular negative is expressed in a Venn diagram by placing an X in the area in *H* outside of *I*.

Although both the premise and the conclusion are particular negatives, and although they’re consistent, comparing their Venn diagrams reveals that the conclusion is the converse of the premise (that is, the X’s are in opposite places). Thus the premise doesn’t imply the conclusion, and the argument is *invalid*.

4.4: III, exercise 16:

First, we give our scheme of abbreviation for the two positive terms:

*P*: prescription drugs.

*A*: medicines with adverse effects.

Our corresponding predicates are:

*Px*: *x* is a prescription drug.

*Ax*: *x* is a medicine with adverse effects.

**Premise** (step 1)**: No non-*P* are non-*A*.**

Step 2: This is expressed in predicate logic as “(*x*)(~*Px* ⊃ ~~*Ax*)”.

Step 3: Applying double negation to “~~*Ax*” gives us “(*x*)(~*Px* ⊃ *Ax*)”.

Step 4: This is the standard way in predicate logic to express the contrapositive universal negative “All non-*P* are *A*.”

Step 5: Our contrapositive universal negative is expressed in a Venn diagram by shading the area outside of both *P* and *A*.

**Conclusion** (step 1): **No *A* are *P*.**

This is already the standard way to express in categorical logic a universal negative, so we skip ahead to step (5).

Step 5: This universal negative is expressed in a Venn diagram by shading the overlap between *A* and *P*.

Comparing the Venn diagrams for the premise and the conclusion reveals that, although the premise and the conclusion are consistent, the premise doesn’t imply the conclusion. Hence the argument is *invalid*.

4.4: III, exercise 19:

First, we give our scheme of abbreviation for the two positive terms:

*P*: pleasant experiences.

*L*: things we like to remember.

Our corresponding predicates are:

*Px*: *x* is a pleasant experience.

*Lx*: *x* is something we like to remember.

**Premise** (step 1)**: All non-*P* are non-*R*.**

Step 2: This is expressed in predicate logic as “(*x*)(~*Px* ⊃ ~*Rx*)”.

Step 3: Applying transposition to “(~*Px* ⊃ ~*Rx*)” gives us “(*x*)(*Rx* ⊃ *Px*)”.

Step 4: This is the standard way in predicate logic to express the universal affirmative “All *R* are *P*.”

Step 5: This universal affirmative is expressed in a Venn diagram by shading the area in *R* outside of *P*.

**Conclusion** (step 1): **All *R* are *P*.**

Since this is already the standard way to express in categorical logic a universal affirmative, we skip ahead to step (5).

In fact, in order to determine whether the argument is valid, we don’t actually have to draw a Venn diagram for the conclusion. This is because the premise and the conclusion are logically equivalent. Hence the argument is *valid*.

**Commutativity:**

As for the operation of **commutativity**, we use it to see that some particular statements are logically equivalent to certain other particular statements. All that commutativity means is that a conjunction is logically equivalent to the conjunction we get when we reverse the two conjuncts. For example, “It’s sunny and it’s warm” is logically equivalent to “It’s warm and it’s sunny.”

In the case of particular affirmatives, for example, we can see that “Some animals are dogs” is logically equivalent to “Some dogs are animals.” This can be seen by expressing these two statements in predicate logic. “Some animals are dogs” gets expressed as “(∃*x*)(*Ax* ⋅ *Dx*).” We then apply commutativity to “(*Ax* ⋅ *Dx*)”, giving us “(∃*x*)(*Dx* ⋅ *Ax*)”, i.e., “Some dogs are animals.”

In the case of particular negatives, for example, we can see that “Some unpleasant tasks are worthwhile” is logically equivalent to “Some worthwhile tasks are not pleasant.” This can be seen by expressing these two statements in predicate logic. “Some unpleasant tasks are worthwhile” gets expressed as “(∃*x*)(*~Px* ⋅ *Wx*).” We then apply commutativity to “(*~Px* ⋅ *Wx*)”, giving us “(∃*x*)(*Wx* ⋅*~Px*)”, i.e., “Some worthwhile tasks are not pleasant.”

Here’s an example of how to apply both commutativity and double negation:

“Some unpleasant tasks are not worthless” = “Some non-*P* are not non-*W*” = (∃*x*)(*~Px* ⋅ *~~Wx*).

Applying double negation to “*~~Wx*” gives us “(∃*x*)(*~Px* ⋅ *Wx*).”

Applying commutativity to “(*~Px* ⋅ *Wx*)” gives us “(∃*x*)(*Wx* ⋅ *~Px*).”

This is the standard way in predicate logic to express “Some *W* are not *P*,” i.e., “Some worthwhile tasks are not pleasant.” Thus this is logically equivalent to our original statement.